Sec. 12.5:
Equations of Lines and Planes

What We Will Go Over In Section 12.5

1. How to find the equation of a line in 3-dim
2. How to find the equation of a plane
3. 3-dim geometry problems involving lines and planes

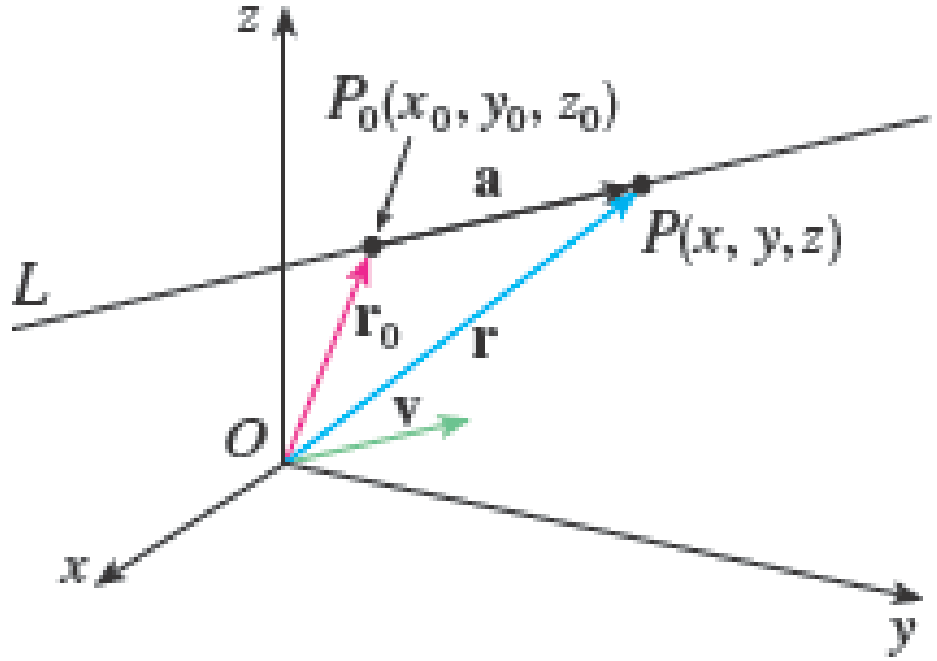
1. How to find the equation of a line in 3-dim

Q: What information do you need to know in order to find the equation of a line?

A: 1) A point on the line

AND

2) the direction (vector) of the line



1. How to find the equation of a line in 3-dim

Equation of a line: If $P(x_0, y_0, z_0)$ is a point on the line and $\vec{v} =< a, b, c >$ is a direction vector for the line and you let $\vec{r}_0 =< x_0, y_0, z_0 >$ be the position vector of point P, then the equation of the line is…

$\vec{r}(t) = \vec{r}_0 + t\vec{v}, \quad t \in \mathbb{R}$ (vector equation)

$x = x_0 + at, \; y = y_0 + bt, \; z = z_0 + ct, \quad t \in \mathbb{R}$
(parametric equations)

$\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$ (symmetric equation)

1. How to find the equation of a line in 3-dim

Quickies:

1) A line passes through the point P(4, -2, 5) and has direction vector $\vec{v} =< -2, 1, 3 >$.

Find the equation of the line in all 3 forms.

1. How to find the equation of a line in 3-dim

Quickies:

2) A line is in the direction of vector $< 4, 0, -2 >$ and passes through the point (-9,-7, 3).

Find the equation of the line in all 3 forms.

1. How to find the equation of a line in 3-dim

Ex 1: A line passes through the point P(2, 5, -1) and has direction vector $\vec{v} =< 6, -3, 7 >$.

a) Find the equation of this line in all 3 forms
b) Find 2 other points on the line other than P
c) Is the point (-10, 11, -15) on the line? What about the point (8, 2, 8)?

1. How to find the equation of a line in 3-dim

Ex 2: A line passes through points P(7, -1, 4) & Q(-5, -6, 2). Find the equation of this line.

1. How to find the equation of a line in 3-dim

Q: What changes if you are trying to find the equation of a line **segment**?

Equation of a line **segment**:

Let $P = (x_0, y_0, z_0)$ and $Q = (x_1, y_1, z_1)$.

Let $\vec{r}_0 =< x_0, y_0, z_0 >$ and $\vec{r}_1 =< x_1, y_1, z_1 >$ be the position vectors of points P and Q.

Then the equation of the line **segment** from P to Q is …

$\vec{r}(t) = (1-t)\vec{r}_0 + t\vec{r}_1, \quad t \in [0,1]$ (vector equation)

1. How to find the equation of a line in 3-dim

Ex 3:

a) Find the equation of segment $\overline{PQ}$ if $P = (2, 2, 5)$ and $Q = (5, -1, -4)$

b) Is the point (8, -4, -13) on the line segment?

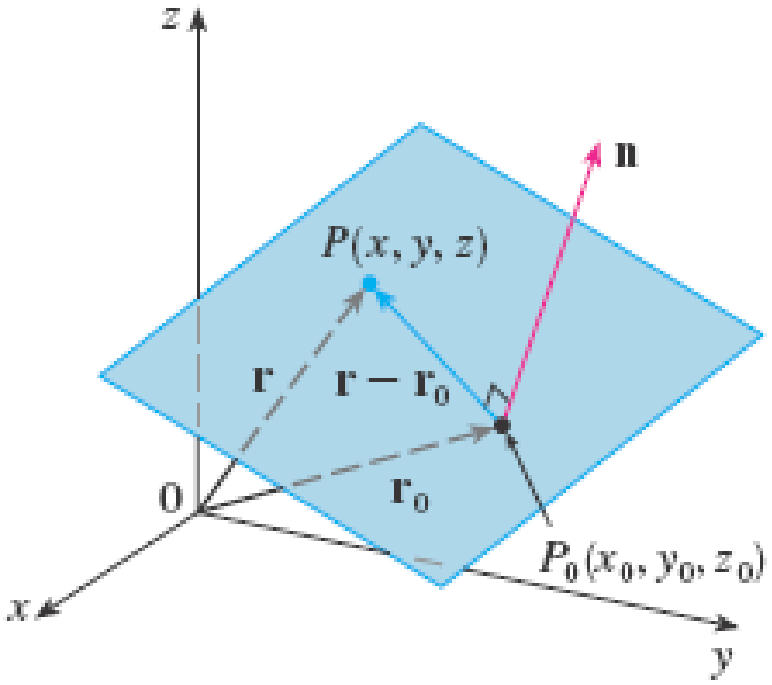
2. How to find the equation of a plane

Q: What information do you need to know in order to find the equation of a plane?

A: 1) A point on the plane

AND

2) a normal vector to the plane



2. How to find the equation of a plane

Equation of a plane: Suppose $P(x_0, y_0, z_0)$ is a point on the plane and $\vec{n} =< a, b, c >$ is a normal vector to the plane. If you let $\vec{r}_0 =< x_0, y_0, z_0 >$ be the position vector of point P and let $\vec{r} =< x, y, z >$ be the position vector of a general point on the plane, then the equation of the plane is…

$$\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0 \qquad \text{or} \qquad \vec{n} \cdot \vec{r} = \vec{n} \cdot \vec{r}_0$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

or

$$ax + by + cz = d \quad \text{where} \quad d = ax_0 + by_0 + cz_0$$

(scalar equation)

2. How to find the equation of a plane

Quickies:

3) Find the equation of the plane that passes through point $P(4,7,-2)$ and has normal vector $\vec{n} =< 2,-4,-1 >$.

2. How to find the equation of a plane

Quickies:

4) Find the equation of the plane that has $< -2, 9, 5 >$ as a normal vector and passes through point $(3, 1, -2)$.

2. How to find the equation of a plane

Ex 4:

a) Find the equation of the plane that passes through points $P(1,2,3)$, $Q(5,-4,2)$, and $R(0,1,-1)$.

b) Is the point (7, -2, -1) on this plane?

c) Find a point on this plane other than points P , Q , and R.

3. 3-dim geometry problems involving lines & planes

Ex 5: Consider the 4 lines and 3 planes given below…

Line 1: $x = 6 + 3t \ , \ y = -2 + t \ , \ z = 3 - 3t \ , \ t \in \mathbb{R}$
Line 2: $x = 3 - 3s \ , \ y = 2 - 2s \ , \ z = 2s \ , \ s \in \mathbb{R}$
Line 3: $x = 9 \ , \ y = 1 + 5u \ , \ z = 4 + 3u \ , \ u \in \mathbb{R}$
Line 4: $x = 1 + 9v \ , \ y = 3 + 6v \ , \ z = 2 - 6v \ , \ v \in \mathbb{R}$
Plane 1: $-2x + 4y + z = 13$
Plane 2: $4x - 5y + 10z = 20$
Plane 3: $x + 2y - 2z = 6$

a) Where does Line 1 cross each of the coordinate planes?
b) Where does Line 2 cross each of the coordinate axes?
c) Where do Lines 1 and 2 intersect?
d) Are Lines 2 and 3 the same line, lines that intersect in only 1 point, parallel lines, or skew lines?
e) Where do Line 1 and Plane 1 intersect?

3. 3-dim geometry problems involving lines & planes

Ex 5: Consider the 4 lines and 3 planes given below…

Line 1: $x = 6 + 3t \ , \ y = -2 + t \ , \ z = 3 - 3t \ , \ t \in \mathbb{R}$
Line 2: $x = 3 - 3s \ , \ y = 2 - 2s \ , \ z = 2s \ , \ s \in \mathbb{R}$
Line 3: $x = 9 \ , \ y = 1 + 5u \ , \ z = 4 + 3u \ , \ u \in \mathbb{R}$
Line 4: $x = 1 + 9v \ , \ y = 3 + 6v \ , \ z = 2 - 6v \ , \ v \in \mathbb{R}$
Plane 1: $-2x + 4y + z = 13$
Plane 2: $4x - 5y + 10z = 20$
Plane 3: $x + 2y - 2z = 6$

f) Find the intercepts of Plane 2 and use them to sketch the graph of plane 2.
g) Find the intersection of Plane 2 with each of the coordinate planes.
h) Find the intersection of Plane 1 and Plane 2.
i) Find the equation of the line that passes through $P(2, 1, -4)$ and is parallel to Line 1.

3. 3-dim geometry problems involving lines & planes

Ex 5: Consider the 4 lines and 3 planes given below…

Line 1: $x = 6 + 3t \ , \ y = -2 + t \ , \ z = 3 - 3t \ , \ t \in \mathbb{R}$
Line 2: $x = 3 - 3s \ , \ y = 2 - 2s \ , \ z = 2s \ , \ s \in \mathbb{R}$
Line 3: $x = 9 \ , \ y = 1 + 5u \ , \ z = 4 + 3u \ , \ u \in \mathbb{R}$
Line 4: $x = 1 + 9v \ , \ y = 3 + 6v \ , \ z = 2 - 6v \ , \ v \in \mathbb{R}$
Plane 1: $-2x + 4y + z = 13$
Plane 2: $4x - 5y + 10z = 20$
Plane 3: $x + 2y - 2z = 6$

j) Find the equation of any line (there are many of them) that passes through $P(2, 1, -4)$ and is perpendicular to Line 1.
k) Find the equation of the plane that passes through $P(-3, 5, -1)$ and is parallel to Plane 2.
l) Find the equation of any plane (there are many of them) that passes through $P(-3, 5, -1)$ and is perpendicular to Plane 2.

3. 3-dim geometry problems involving lines & planes

Ex 5: Consider the 4 lines and 3 planes given below…

Line 1: $x = 6 + 3t \ , \ y = -2 + t \ , \ z = 3 - 3t \ , \ t \in \mathbb{R}$
Line 2: $x = 3 - 3s \ , \ y = 2 - 2s \ , \ z = 2s \ , \ s \in \mathbb{R}$
Line 3: $x = 9 \ , \ y = 1 + 5u \ , \ z = 4 + 3u \ , \ u \in \mathbb{R}$
Line 4: $x = 1 + 9v \ , \ y = 3 + 6v \ , \ z = 2 - 6v \ , \ v \in \mathbb{R}$
Plane 1: $-2x + 4y + z = 13$
Plane 2: $4x - 5y + 10z = 20$
Plane 3: $x + 2y - 2z = 6$

m) Find the distance between $P(1, -3, 2)$ and Line 2.
n) Find the distance between $P(1, -3, 2)$ and Plane 2.
o) Find the distance between Plane 2 and Plane 3.
p) Find the distance between Line 1 and Line 2.
q) Find the angle between Plane 1 and Plane 2.

3. 3-dim geometry problems involving lines & planes

Ex 5: Consider the 4 lines and 3 planes given below…

Line 1: $x = 6 + 3t \ , \ y = -2 + t \ , \ z = 3 - 3t \ , \ t \in \mathbb{R}$
Line 2: $x = 3 - 3s \ , \ y = 2 - 2s \ , \ z = 2s \ , \ s \in \mathbb{R}$
Line 3: $x = 9 \ , \ y = 1 + 5u \ , \ z = 4 + 3u \ , \ u \in \mathbb{R}$
Line 4: $x = 1 + 9v \ , \ y = 3 + 6v \ , \ z = 2 - 6v \ , \ v \in \mathbb{R}$
Plane 1: $-2x + 4y + z = 13$
Plane 2: $4x - 5y + 10z = 20$
Plane 3: $x + 2y - 2z = 6$

r) Are Lines 3 and 4 parallel, perpendicular, or neither?
s) Are Lines 2 and 4 parallel, perpendicular, or neither?